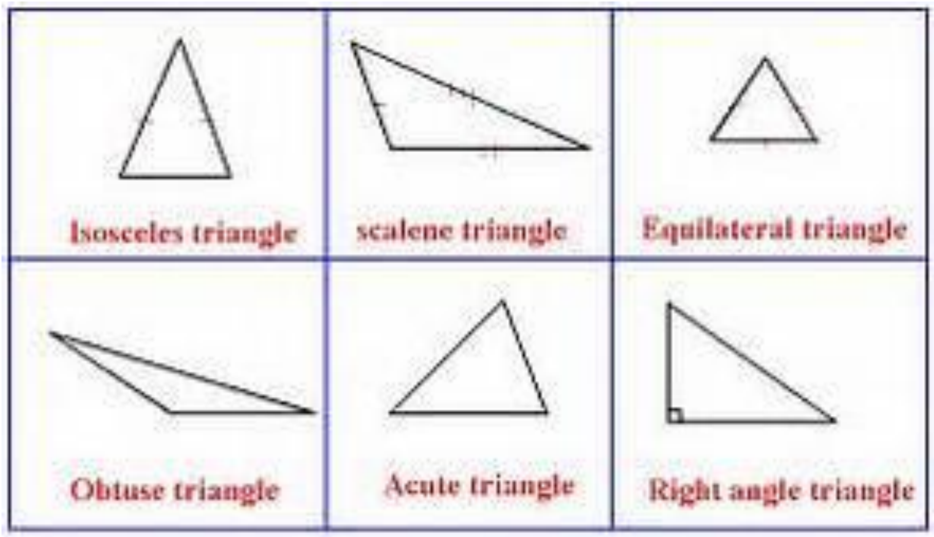
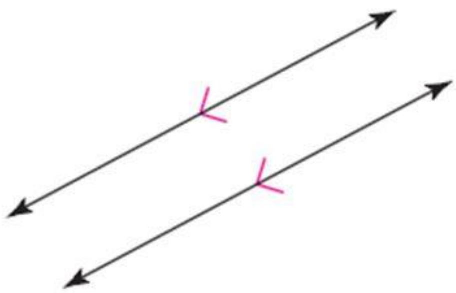
Geometry Placement Practice

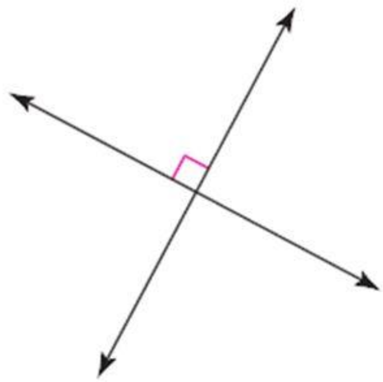
This document provides common geometric formulas and practice problems with solutions to prepare for placement test for Geometry at Heartland Community College. The questions are from the COMPASS website, © 2014 ACT, Inc.

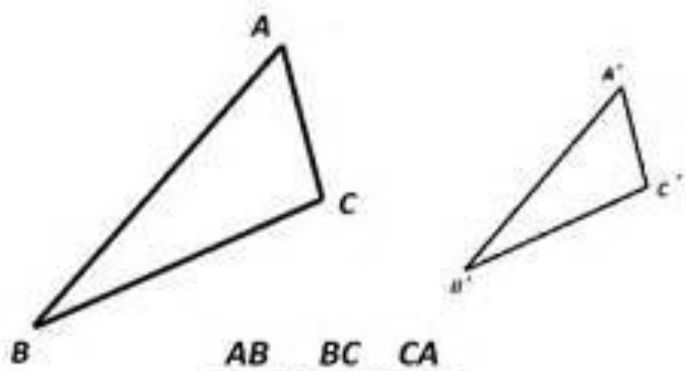


Parallel lines are lines in the same plane that never intersect. They are always the same distance apart.

Perpendicular lines are lines that meet at a right angle, that is, at an angle that measures 90°.

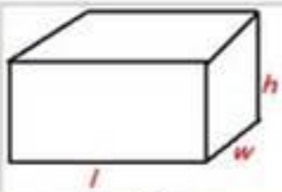
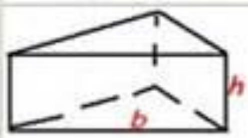
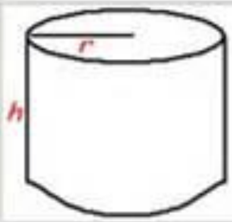






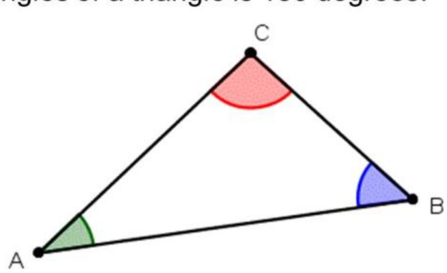
$$\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CA}{C'A'}$$

Term	Dimensions	Graphic	Symbol
Point	Zero		•A
Line Segment	One		$\overline{AB}$
Ray	One		$\overrightarrow{AB}$
Line	One		$\overleftrightarrow{AB}$
Plane	Two		Plane M

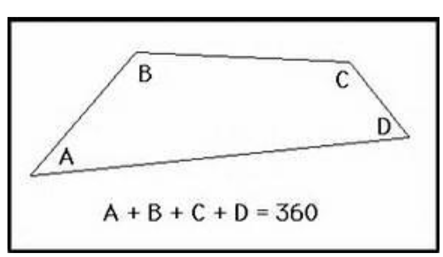
	**Rectangular Solid** Volume = Length X Width X Height V = lwh Surface = 2lw + 2lh + 2wh
	**Prisms** Volume = Base X Height v=bh Surface = 2b + Ph *(b is the area of the base P is the perimeter of the base)*
	**Cylinder** Volume = $\pi r^2 \times$ height V = πr^2 h Surface = 2π radius x height S = $2\pi rh + 2\pi r^2$

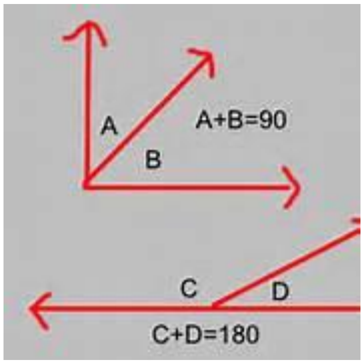
Triangle Sum Theorem

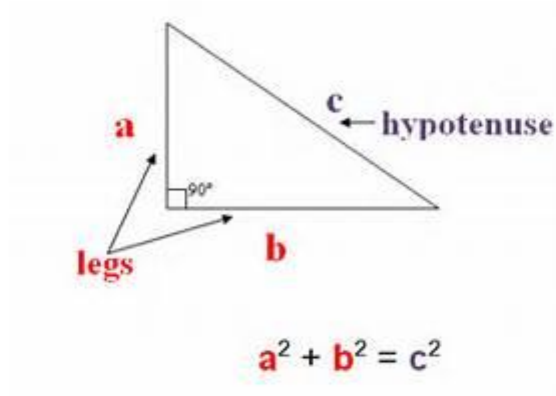
- The sum of the interior measures of the angles of a triangle is 180 degrees.



$$m\angle A + m\angle B + m\angle C = 180$$



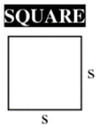




In the image above on the left, angles A and B are complementary, or complements of one another, because the sum of their measures is 90 degrees. Angles C and D are supplementary, or supplements of one another, because the sum of their measures is 180 degrees. Angles C and D are also a linear pair since they form a line.

PERIMETER & AREA FORMULAS

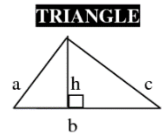
SQUARE



$P = 4s$

$A = s^2$

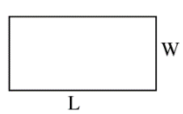
TRIANGLE



$P = a + b + c$

$A = \frac{1}{2}bh$ **OR** $A = \frac{bh}{2}$

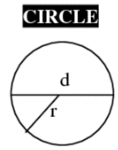
RECTANGLE



$P = 2L + 2W$

$A = LW$

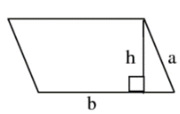
CIRCLE



$C = \pi d$ **OR** $C = 2\pi r$

$A = \pi r^2$

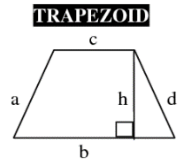
PARALLELOGRAM



$P = 2a + 2b$

$A = bh$

TRAPEZOID

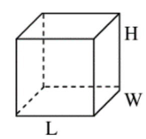


$P = a + b + c + d$

$A = \frac{1}{2}h(b + c)$

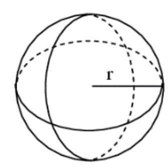
VOLUME FORMULAS

RECTANGULAR SOLID



$V = LWH$

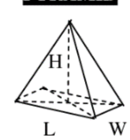
SPHERE



$V = \frac{4}{3}\pi r^3$ **OR**

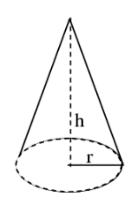
$V = \frac{4\pi r^3}{3}$

RECTANGULAR PYRAMID



$V = \frac{1}{3}LWH$ **OR** $V = \frac{LWH}{3}$

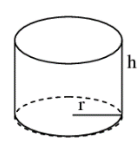
RIGHT CIRCULAR CONE



$V = \frac{1}{3}\pi r^2 h$

OR $V = \frac{\pi r^2 h}{3}$

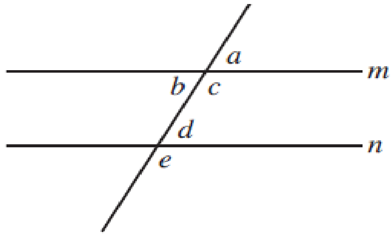
RIGHT CIRCULAR CYLINDAR



$V = \pi r^2 h$

Practice Problems

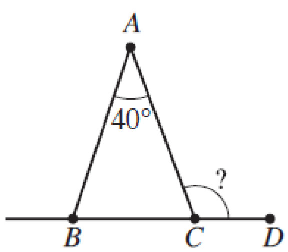
1. In the figure below, line *m* is parallel to line *n*, and line *t* is a transversal crossing both *m* and *n*. Which of the following lists has 3 angles that are all equal in measure?



A. $\angle a, \angle b, \angle d$
B. $\angle a, \angle c, \angle d$
C. $\angle a, \angle c, \angle e$
D. $\angle b, \angle c, \angle d$
E. $\angle b, \angle c, \angle e$

Solution: Since line *m* and line *n* are parallel, then corresponding angles on each side of the transversal are congruent (equal in measure) which means that ∠ a and ∠ d have equal measure. In addition, vertical angles (like ∠ a and ∠b) have equal measure. Therefore, angles a, d, and b all have equal measure.

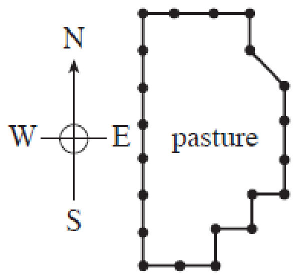
2. As shown in the figure below, ΔABC is isosceles with the length of $\overline{AB}$ equal to the length of $\overline{AC}$. The measure of $\angle BAC$ is 40° and points B, C, and D are collinear. What is the measure of ∠ACD?



A. 70°
B. 80°
C. 110°
D. 140°
E. 160°

Solution: Since the measures of the three angles in any triangle add up to 180 degrees, we know that the measures of ∠ABC and ∠ACB will add to be 140 degrees (180° – 40°). Also, ∠ABC and ∠ACB have equal measure since an isosceles triagle has two congruent angles. Therefore, each measures 140/2 , or 70°. Finally, ∠ACB and ∠ACD are a linear pair (form a line) and their measures will add to 180 degrees. Therefore, the missing angle measure is 180° – 70° = 110°.

3. The diagram below shows a pasture which is fenced in. All but one section of fence run straight north-south or east-west. Consecutive fence posts are 10 feet apart except for the one diagonal section. Which of the following statements best describes P, the perimeter of the pasture, in feet?



A. $P > 210$
B. $P = 210$
C. $P < 210$
D. $P > 230$
E. $P = 240$

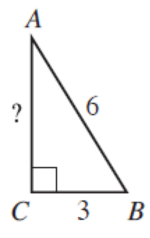
Solution: Counting all lengths between fence posts, we have 7 vertical lengths on the left side, 6 vertical lengths on the right side, 3 horizontal lengths on the top and 4 horizontal lengths on the bottom. This gives us 20 lengths or 200 feet of fencing. Using the Pythagorean Theorem, with two base sides of 10, we can calculate that the diagonal is $10\sqrt{2}$. The $\sqrt{2}$ is approximately 1.4 giving an approximation of (10)(1.4) = 14 for the length of the diagonal side. Thus, the perimeter measures around 214 feet. Therefore, the perimeter is greater than 210, but less than 230.

4. A person had a rectangular-shaped garden with sides of lengths 16 feet and 9 feet. The garden was changed into a square design with the same area as the original rectangular-shaped garden. How many feet in length are each of the sides of the new square-shaped garden?

A. 7
B. 9
C. 12
D. $5\sqrt{7}$
E. 16

Solution: The area of the original rectangular-shaped garden was A = 16 * 9 = 144 ft^2. For a square with sides of length s, the area is s^2. In this case we need the area of the square to equal the area of the rectangle, or $s^2 = 144ft^2$. Taking the positive square root of both sides we get s = 12 feet for each side length of the new square-shaped garden.

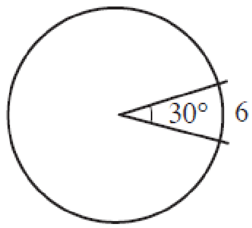
5. In the figure below, ΔABC is right triangle. The length of $\overline{AB}$ is 6 units and the length of $\overline{CB}$ is 3 units. What is the length, in units, of $\overline{AC}$?



A. 5
B. $3\sqrt{3}$
C. $3 + \sqrt{5}$
D. $3\sqrt{5}$
E. $3\sqrt{6}$

Solution: Using the Pythagorean Theorem, we have $3^2 + ?^2 = 6^2$. Solving this, we have $9 + ?^2 = 36$ or $?^2 = 27$. Taking the positive square root of both sides, we get $\sqrt{27}$=$\sqrt{9 * 3} = 3\sqrt{3}$ for the missing side length.

6. If a central angle of measure 30° is subtended by a circular arc of length 6 meters, as is illustrated below, how many meters in length is the radius of the circle?



A. $\frac{\pi}{36}$
B. $\frac{1}{5}$
C. π
D. $\frac{36}{\pi}$
E. 180

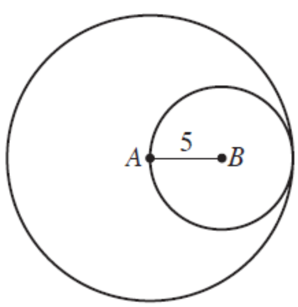
Solution: We know that a central angle of 360° is subtended by a circular arc of length $2\pi r$. Using proportions, we have $\frac{2\pi r}{360} = \frac{6}{30}$. To solve for the radius, we need to isolate *r*. Reduce both fractions first to get $\frac{\pi r}{180} = \frac{1}{5}$. Finally, multiply both sides by $\frac{180}{\pi}$ to get $r = \frac{36}{\pi}$.

7. A rectangular box with a base 2 inches by 6 inches is 10 inches tall and holds 12 ounces of breakfast cereal. The manufacturer wants to use a new box with a base 3 inches by 5 inches. How many inches tall should the new box be in order to hold exactly the same volume as the original box? (Note: The volume of a rectangular box may be calculated by multiplying the area of the base by the height of the box.)

A. 8
B. 9
C. 10
D. 11
E. 12

Solution: The volume of the original rectangular box is $2 \cdot 6 \cdot 10$ (V = LWH), or V = 120 in^3. This holds 12 ounces of cereal. We want to know the height of a new box that will also hold 12 ounces of cereal or have volume of 120 in^3. This provides the equation: 120 = 3 · 5 · H. Solving for H, we have H = 8 inches.

8. In the figure below, the circle centered at B is internally tangent to the circle centered at A. The smaller circle passes through the center of the larger circle and the length of $\overline{AB}$ is 5 units. If the smaller circle is cut out of the larger circle, how much of the area, in square units, of the larger circle will remain?

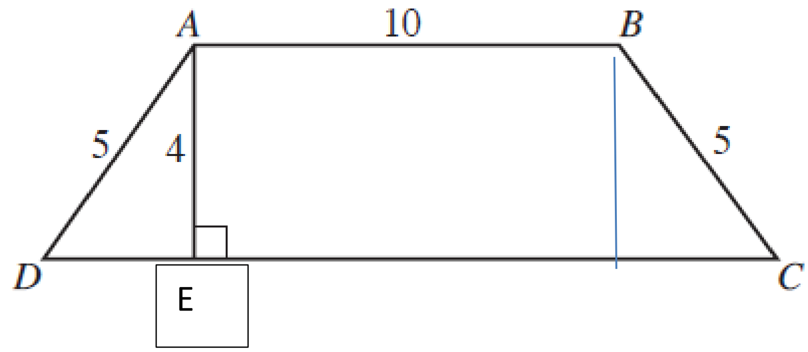


A. 10π
B. 25π
C. 75π
D. 100π
E. 300π

Solution: This problem consists of finding the area of the larger circle and subtracting the area of the smaller circle from it. The radius of the smaller circle is 5 and the radius of the larger circle is 10 (since the length from point B to the outer part of the circle is 5 and the length of $\overline{AB}$ is also 5.) Therefore, the area of the large circle is 100π and the area of the small circle is 25π. Their difference is $100\pi - 25\pi = 75\pi$.

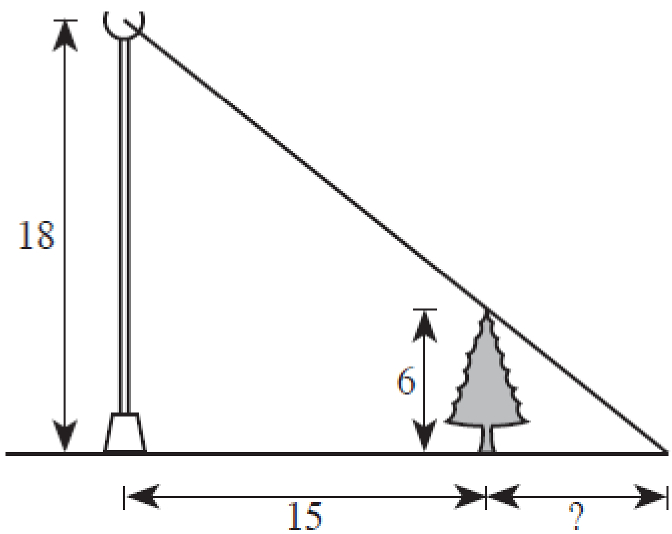
9. In the figure below, $\overline{AB}$ and $\overline{CD}$ are parallel, and lengths are given in units. What is the area, in square units, of trapezoid ABCD?

A. 36
B. 52
C. 64
D. 65
E. 104



Solution: We need to find the area of a trapezoid. The length of $\overline{AB}$ is 10. In order to find the length of $\overline{DC}$, we will need to find DE, the missing length of the side of the right triangle. We will use the Pythagorean Theorem to do that. Our equation is $4^2 + DE^2 = 5^2$. 16 + DE^2 = 25 or $DE^2 = 9$. $\overline{DE}$ has length 3. Notice that we have that triangle on both the left and right sides of this trapezoid giving $\overline{DC}$ a length of 10 + 3 + 3 or 16. The area of the trapezoid is A = $\frac{1}{2}$(4)(16 +10) = 2(26) = 52 square units.

10. A 6-foot spruce tree is planted 15 feet from a lighted streetlight whose lamp is 18 feet above the ground. How many feet long is the shadow of that tree?



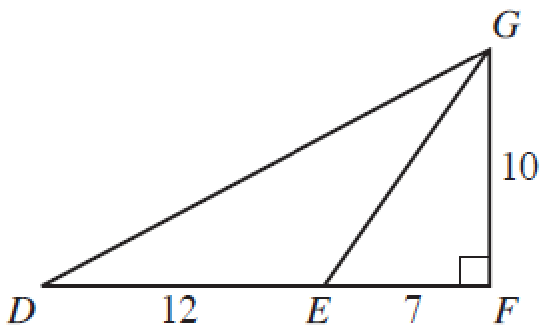
A. 5.0
B. 7.5
C. 7.8
D. 9.6
E. 10.0

Solution: We will use similarity of triangles in this problem. Note that the large triangle has base of length 15 + ? or 15 + x and height of 18. Since these are similar triangles (all angles have the same measure), the ratio of corresponding sides will be equal. This gives us the equation

$\frac{18}{15+x} = \frac{6}{x}$.

Using cross multiplication, we have 18x = 6(15 + x).
Distributing the 6 yields 18x = 90 + 6x
12x = 90
x = 7.5, the shadow is 7.5 feet long.

11. In the figure below, the lengths of $\overline{DE}$, $\overline{EF}$, and $\overline{FG}$ are given, in units. What is the area, in square units, of ΔDEG?

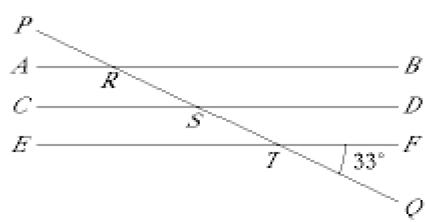


A. 29
B. 47.5
C. 60
D. $6\sqrt{149}$
E. 120

Solution: Our strategy is to take the area of ⊿GFD and subtract the area of ⊿GFE since both of these are right triangles with area, A = $\frac{1}{2}bh$. The area of $\measuredangle GFD$ is $\frac{1}{2}(19 * 10) = 95$ *square units*. The area of $\measuredangle GFE$ is $\frac{1}{2}(7 * 10) = 35$ *square units*. Therefore the area of ΔDEG is 95 – 35 or 60 *square units.*

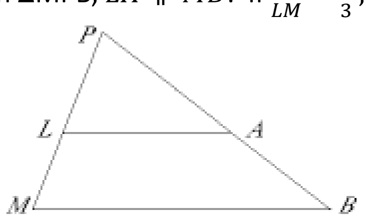
12. In the figure below $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$ are parallel, and $\overline{PQ}$ intersects all 3 lines at points R, S, and T, respectively. If the measure of ∠QTF is 33°, what is the measure of ∠PRB ?

A. 33°
B. 57°
C. 66°
D. 123°
E. 147°



Solution: When parallel lines are cut by a transversal, alternate interior angles are congruent. In this case, that means m∠ PRA is 33°. Since ∠ PRA and ∠ PRB are a linear pair, the sum of their measures is 180°. Thus, m∠ PRB is 180° – 33° or 147°.

13. In ΔMPB, $\overline{LA} \parallel \overline{MB}$. If $\frac{PL}{LM} = \frac{5}{3}$, find $\frac{PB}{PA}$.



A. $\frac{5}{8}$ B. $\frac{2}{3}$ C. $\frac{8}{5}$ D. $\frac{5}{3}$ E. $\frac{8}{3}$

Solution: When parallel lines are cut by a transversal, the ratios of corresponding lengths are equal. Therefore, $\frac{PB}{PA} = \frac{PM}{PL}$. We are given, $\frac{PL}{LM} = \frac{5}{3}$. Using line segment addition, we have PL + LM = PM. Therefore, $\frac{PM}{PL} = \frac{PL+LM}{PL} = \frac{5+3}{5} = \frac{8}{5}$. Thus, $\frac{PB}{PA} = \frac{8}{5}$.

Answers:

1. **A**
2. **C**
3. **A**
4. **C**
5. **B**
6. **D**
7. **A**
8. **C**
9. **B**
10. **B**
11. **C**
12. **E**
13. C